An Approximation for Routing Planning, Mobile Charging, and Energy Sharing for Sensing Devices

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***Abstract*—Wireless Charging Vehicles (WCVs) have been widely explored as a means of enabling continuous operation of sensors that are powered by batteries. However, the energy consumption of WCVs can be inefficient, leading to insufficient energy supply for sensors that are located in challenging-to-access areas. Consequently, there is a need to design an effective charging and energy sharing scheme for sensors to improve the quality of service in this setup. This paper focuses on the Joint Optimization of Mobile charging and Energy sharing of sensors (JOIN-ME) problem, which is known to be NP-hard. To address this challenge, we first transform JOIN-ME into a submodular maximization problem with general constraints. Subsequently, we propose the Routing planning, Mobile charging, and Energy sharing for Sensing devices (RMES) algorithm, which has an approximation ratio of $1/8(1-1/e)$. Finally, we conduct experiments to showcase the superior performance of RMES compared to existing baselines, under varying scales and constraints. Our work on the design of an efficient charging and energy sharing scheme for sensors can significantly improve the reliability and longevity of wireless sensor networks, enabling the deployment of these networks in critical applications such as environmental monitoring, crowd sensing, and security surveillance.**

***Index Terms*—Mobile Charging, Energy Sharing, Charging Route Planning, Sensing Devices, Approximation Algorithm**

I. Introduction

With the rapid development of sensing devices, Wireless Sensor Network (WSN) has been widely deployed and applied in recent years [1]–[5]. In WSN, each sensor needs to be recharged by power supplies to sustain the normal operation. Traditional recharging is mostly based on wired charging. At the same time, recent breakthrough in Wireless Power Transfer technology (WPT) [6] allows wireless energy transfer from power supplies to rechargeable sensing devices, providing a promising and flexible recharging way called mobile charging and leading to the emergence of the Wireless Rechargeable Sensor Network (WRSN), where the WPT technology is applied in WSN. In the general scenario of mobile charging, one or multiple Wireless Charging Vehicles (WCVs) periodically travel in WRSN to charge rechargeable sensors to ensure them work perpetually. So far, a lot of research efforts [7], [8] have been devoted to the mobile charging in WRSN and some other related applications in body area network [9], underwater monitoring [10], coal mining [11] and so on.

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Fig. 1. Mobile charging with energy sharing in WRSN

In the early years, as the effective charging range of WCVs is quite finite (typically as low as a few meters in practice) and sensors are physically isolated from each other, WCVs need to move to sensors close to charge the sensors. Most works are based on such a scenario [12], [13]. The challenges of mobile charging in Wireless Rechargeable Sensor Networks include energy and time limitations of the Wireless Charging Vehicles, energy overheads caused by their travelling, and remote locations of some sensors.

To alleviate the dilemma, Zhu et al. [14], [15] proposed the energy sharing technology, which allows sensors to share energy with others. In [16], the technology is utilized to cope with harsh energy propagation conditions in underground communications. However, related works do not consider planning WCVs' charging routes.

A feasible solution for the mobile charging problem is combining route planning with energy sharing. The WCV can visit and charge only selected sensors, while the charged sensors can share their energy with other sensors through single-hop or multi-hop energy transfer. This approach provides energy and time-efficient routes and can successfully charge even hard-to-access sensors with auxiliary devices. Though some energy waste may occur during sharing, it is expected to be less than the conservation of WCV. Necessary devices require extra cost but can be amortized over time.

The JOIN-ME challenge tackles the optimization of the charging strategy and path planning for mobile charging of

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sensing devices. Challenges include energy sharing, multi-charging choices, and charging strategy and path planning. A greedy energy sharing strategy is used to address these challenges by transforming the joint optimization problem into a submodular maximization problem. The proposed RMES algorithm has an approximation ratio of $1/8(1-1/e)$, outperforming baseline algorithms, and being efficient enough for real-world applications.

The main contributions of this paper are as follows:

- We formulate the JOIN-ME problem into a submodular maximization problem with general constraints.
- We design an approximation algorithm called Routing planning, Mobile charging and Energy sharing for Sensing devices(RMES) algorithm with an approximation ratio of $\frac{1}{8}(1-1/e)$. We also used a learning based algorithm to solve the TSP problem and get the cost of the WCV.
- We conduct simulation experiments to evaluate RMES. The performance of the RMES problem is evaluated by changing the WCV energy limit and the sensor number. We also choose some baselines to compare with our algorithm.

II. Charging Model and Problem Formulation

A. Charging Model Description

We consider a WRSN consisting of n rechargeable sensors, a WCV, and a base station. The WCV is the energy supplier for all sensors in the network, whether directly or indirectly. We assume that the WCV visits several sensors periodically to maintain the normal operation of the sensor network. The WCV starts from the base station for each tour and visits several sensors. The sensors not visited by the WCV gain energy from the sensors charged by WCV. Finally, the WCV finishes this charging tour and goes back to the base station.

Consider a set of sensors $S = \{s_1, s_2, \cdots, s_n\}$, which distribute in a 2D $d \times d$ plane. The sensors have diverse energy demands $E = \{e_1, e_2, \cdots, e_n\}$.

There are two ways for a sensor to gain energy:

a) **WCV charging:** A subset of sensors are directly charged by the WCV. Denote the direct energy gain from the WCV as $E_c = \{x_1, x_2, \cdots, x_n\}$, where x_i is the energy gain of sensor s_i from WCV and the charging set is $X = \{s_i \mid x_i > 0\}$. Since a long time of charging may cause the sensor to overheat, we assume that every sensor can only be charged by the WCV with a limited energy $E_{\max}$.

We also assume that the WCV has a limited energy $C_{\max}$, and the cost of the WCV is a constraint for the WRSN. There are two energy costs for WCV charging: the travelling energy cost and the charging energy cost. For travelling energy cost, we can denote that $\alpha L(T)$, where α is the energy cost for the WCV travel a unit distance, and $L(T)$ is the distance for the WCV to visit all the charging points of T. For charging energy cost, we can denote that $\sum_{i=1}^{n} x_i/\eta_i = \sum_{i=1}^{n} T_i e_{\min}/\eta_i$, where η_i is the charging efficiency of the WCV for sensor s_i. We can get the total energy cost of WCV charging as follows:

$$C(\mathbf{T}) = \alpha L(\mathbf{T}) + \sum_{i=1}^{n} T_i e_{\min}/\eta_i. \tag{1}$$

Here we use the euclidean distance to calculate $L(\mathbf{T})$, which is less than the actual distance. In the simulation, we will give a bigger α to offset the impact.

The total energy cost of WCV has a limit $C_{\max}$, which is the maximum energy cost the WCV can afford before it returns to the base station.

b) **Energy sharing:** Network connections between sensors are common because data transmission and aggregation between sensors are needed. These networks can also be reused as energy transmission networks. Inherently, energy loss is inevitable during sharing and is closely related to practical factors such as distance and the transmission cable. We use η_{ij} to denote the energy efficiency from sensor s_i to sensor s_j. If s_i cannot directly charge s_j, then $\eta_{ij} = 0$. In our study, we define η_{ij} decreasing linearly:

$$\eta_{ij} = \begin{cases} \max\{1 - pd_{ij}, 0\} & \text{if } s_i \text{ can charge } s_j, \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where p is the energy loss coefficient, and d_{ij} is the distance between s_i and s_j. This energy efficiency definition is only a simple example of sharing efficiency definition, and our algorithm can be extended to more complex cases.

Notice that the energy sharing process can be a multi-hop process. η_{ij} may not be the max energy efficiency from s_i to s_j, because probably there is a sensor s_k between s_i and s_j, and $\eta_{ik}\eta_{k_j} > \eta_{ij}$. We use $\eta_{ij}^{\max}$ to denote the max energy efficiency from s_i to s_j.

Suppose the energy transmitted from sensor s_i to sensor s_j is x_{ij}, then the energy received by sensor s_j is $x_{ij}\eta_{ij}^{\max}$. We use E_s to denote the energy sharing vector, i.e., $E_s = \{x_{ij} | i, j = 1, 2, \cdots, n\}$.

After the WCV charging and energy sharing, the sensor s_i gain $x_i = T_i e_{\min}$ from WCV, $\sum_{j=1}^{n} x_{ji}\eta_{ji}^{\max}$ from energy sharing, and gives $\sum_{j=1}^{n} x_{ij}$ to other sensors. Then we get the residual energy $\hat{e_i}$ of sensor s_i as:

$$\hat{e_i} = x_i + \sum_{j=1}^{n} x_{ji}\eta_{ji}^{\max} - \sum_{j=1}^{n} x_{ij}. \tag{3}$$

B. Problem Formulation

In order to improve the overall availability and coverage, we define the energy utility U_i for sensor s_i as follows:

Definition 1 (Energy utility U_i for a sensor)**.** *The energy utility U_i for sensor s_i is defined as*

$$U_i = \min\{\hat{e_i}/e_i, 1\}, \tag{4}$$

where we compare the ratio of the sensor's residual energy $\hat{e_i}$ and its energy requirement e_i with 1, and the smaller of the two is taken as the utility value.

Summing up the energy utility of all sensors, we get the total energy utility U, which is closely related to the energy gain of the WCV charging choice $\mathbf{T}$ and energy sharing E_s:

$$U(\mathbf{T}, E_s) = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} \min\{\hat{e_i}/e_i, 1\}. \tag{5}$$

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Here, We want to maximize the total utility $U(\mathbf{T}, E_s)$ and formulate the problem as follows:

Definition 2 (JOIN-ME problem)**.** *Given a set of sensors S and a WCV with total energy cosr limit $C_{\max}$, Our goal is to choose the best charging choice $\mathbf{T}$ and energy sharing E_s to maximize the total energy utility U. Consequently, we can formulate the problem of Joint OptimIzatioN of Mobile charging and Energy sharing of sensors (JOIN-ME) as follows:*

$$
\begin{aligned}
\max \quad & U(\mathbf{T}, E_s); & & \\
\text{s.t.} \quad & C(\mathbf{T}) \le C_{max}, & & X \subseteq S; \\
& \hat{e}_i \ge 0, & & i = 1, 2, \cdots, n; \\
& x_i \le E_{max}, & & i = 1, 2, \cdots, n; \\
& x_{ij} \ge 0 & & i, j = 1, 2, \cdots, n;
\end{aligned}
\tag{6}
$$

C. NP-Hardness Proof of JOIN-ME

The JOIN-ME problem can be decomposed into two phases, the energy sharing and charging intricately interconnected phases. The energy sharing phase entails developing an energy management strategy that accounts for the energy status of individual sensors and the network's energy requirements, to determine the optimal distribution of energy resources among sensors. The charging phase requires devising a charging strategy for the WCV that maximizes the energy gain of the network while minimizing the charging cost. Notably, the solution to the charging problem depends on the energy sharing phase results. The charging strategy is further evaluated based on cost, which entails resolving the Traveling Salesman Problem (TSP) to determine the optimal charging route.

Theorem 1. *The JOIN-ME problem is NP-hard.*

Proof: The hardness proof is based on the NP-hard *Charging Reward Maximization problem* in [17] where a mobile charger moves and charges sensors in WRSNs such that the sum of charging rewards from all charged sensors, which is proportional to the amount of energy charged, is maximized without violation of charger energy capacity. With all sharing energy variables x_{ij} $(i, j = 1, 2, \cdots, n)$ set to 0, our problem can be directly reduced from the above problem. Therefore, we conclude that JOIN-ME is NP-hard. ■

III. RMES Algorithm

A. Energy Sharing Strategy

In the energy sharing phase of RMES, we assume that the WCV has already chosen a charging choice $\mathbf{T}$ and a subset X of sensors to charge. Then we solve the energy sharing problem to obtain the appropriate energy sharing vector E_s. We first generate the maximum energy efficiency matrix $\eta_{ij}^{\max}$ and then solve the energy sharing problem according to the idea of a greedy algorithm.

We will first generate the maximum energy efficiency matrix $\eta_{ij}^{\max}$, as shown in Lines 1 to 2 of Algorithm 1. This problem can be solved easily by converting it into an all-pairs shortest-path problem. By giving each edge a weight $-\log(\eta_{ij})$, we transform the multiplication operation of efficiency into

Algorithm 1: Greedy Energy Sharing Algorithm

Input: Sensor set S,
$E_c = \{T_1 e_{\min}, T_2 e_{\min}, \cdots, T_n e_{\min}\}$.
Output: Matrix variables of sharing energy E_s.

/* Generate maximum energy efficiency matrix $\eta_{ij}^{\max}$. */
1 Get all-pairs shortest-path matrix $D_{ij}^{\max}$ by Johnson algorithm for edge weight $-\log(\eta_{ij})$;
2 $\eta_{ij}^{\max} \leftarrow \exp(-D_{ij}^{\max})$;
/* Solve energy sharing problem. */
3 Set $x_{ij} \leftarrow 0$ and $\hat{e}_i \leftarrow x_i$ for $i, j = 1, 2, \cdots, n$;
4 $k_{ij} \leftarrow \eta_{ij}^{\max} e_i / e_j$ for all i, j;
5 Sort $K = \{k_{ij}\}$ non-increasingly;
6 **for** $k_{ij} \in K$ **do**
7   **if** $\hat{e_j} < e_j$ **and** $k_i j > 1$ **then**
8     $x_{ij} \leftarrow \min\left\{\dfrac{e_j - \hat{e_j}}{\eta_{ij}^{\max}}, \hat{e_i}\right\}$;
9     $\hat{e_i} \leftarrow \hat{e_i} - x_{ij}$; $\hat{e_j} \leftarrow \hat{e_j} + x_{ij}\eta_{ij}^{\max}$;
10 **return** $E_S = \{x_{ij} | i, j = 1, 2, \cdots, n\}$.

an addition operation in the shortest-path problem. Use the Johnson algorithm to solve the all-pairs shortest-path problem, and suppose the result is $D_{ij}^{\max}$, then we can get the maximum energy efficiency matrix $\eta_{ij}^{\max}$ by $\eta_{ij}^{\max} = \exp(-D_{ij}^{\max})$.

Then we solve the energy sharing problem, which is shown in Lines 2 to 6 of Algorithm 1. We first assume that s_i transmits x_{ij} amount of energy to s_j. The original part of x_{ij} in energy ratio before sharing is x_{ij}/e_i and after sharing it turns into $(\eta_{ij}^{\max} x_{ij})/e_j$ since s_j receives $\eta_{ij}^{\max}$ amount of energy. In order to increase energy ratio, we have $(\eta_{ij}^{\max} x_{ij})/e_j > x_{ij}/e_i$, and we get $\frac{\eta_{ij} e_i}{e_j} > 1$. If energy on s_j is no less than its demand e_j, it means s_j obtains enough energy either from WCV or sensors ahead of s_i in X, s_i will not share energy with s_j.

In overall greedy research, we use the utility increase ratio

$$k_{ij} = \eta_{ij}^{\max} e_i / e_j, \quad i, j = 1 \cdots n, \tag{7}$$

to determine the ratio of utility x_{ij} make before and after the energy transmits from s_i to s_j. We sort the k_{ij} in a non-increasing way to ensure the sharing with higher ratio change can be applied. For any s_i and s_j, s_i only transfer energy when s_j has not reached its demand e_j and $k_{ij} > 1$. After one sensor completes, we access the next sensor in the sequence and perform the same operations until all charged sensors are processed. In this way, we can find an energy sharing vector E_s to maximize the overall energy utility when given a charging subset X and charging energy E_c.

B. Energy Allocation Strategy

In the WCV charging phase of the RMES algorithm, we find the optimal charging subset X and charging energy E_c to maximize the overall energy utility. To solve this problem, we

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Algorithm 2: Cost-Efficient Algorithm

Input: Sensor set S with related properties, max charging energy $E_{\max}$, discretization factor n_T.
Output: Charging choice vector $\mathbf{T}$, energy sharing matrix E_s.

1 $\mathbb{T}^{\text{choice}} \leftarrow \left\{\mathbf{t}_{i,t}^{\text{choice}} \middle| i=1,2,\cdots,n; t=1,2,\cdots,n_T\right\}$;
2 Set T to a zero vector, E_s to a zero matrix;
3 **while** $\mathbb{T}^{choice}$ *is not empty* **do**
4   **for** $\forall \mathbf{t}_{i,t}^{choice} \in \mathbb{T}^{choice}$ **do**
5     $\mathbf{T}'(i,t) \leftarrow \mathbf{T}$; $\mathbf{T}'_i(i,t) \leftarrow t$;
6   Get according energy sharing matrix $E'_s(i,t)$ by Algorithm 1 of choice $\mathbf{T}'$;
7     $\Delta U(i,t) \leftarrow U(\mathbf{T}'(i,t), E'_s(i,t)) - U(\mathbf{T}, E_s)$;
8     $\Delta C(i,t) \leftarrow C(\mathbf{T}'(i,t)) - C(\mathbf{T})$;
9   **if** $C(\mathbf{T}'(i,t)) \leq C_{\max}$ *or* $\Delta U(i,t) = 0$ **then**
10       $\mathbb{T}^{\text{choice}} \leftarrow \mathbb{T}^{\text{choice}} \backslash \mathbf{t}_{i,t}^{\text{choice}}$
11   $\mathbf{t}_{i,t}^{\text{choice}} \leftarrow \arg\max_{\mathbf{t}_{i,t}^{\text{choice}}} \dfrac{\Delta U(i,t)}{\Delta C(i,t)}$ for $\forall \mathbf{t}_{i,t}^{\text{choice}} \in \mathbb{T}^{\text{choice}}$;
12   Set $T \leftarrow T'(i,t)$ and $E_s \leftarrow E'_s(i,t)$;
13   $\mathbb{T}^{\text{choice}} \leftarrow \mathbb{T}^{\text{choice}} \backslash \{\mathbf{t}_{i,j}^{\text{choice}}, j = 1 \cdots t\}$;

propose a cost-efficient algorithm referring to [18] as shown in Algorithm 2. The main idea of the algorithm is to determine the best WCV charging strategy to optimize the utility. In every iteration, we choose the sensor s_i with the highest energy utility ratio $\frac{\Delta U(i,t)}{\Delta C(i,t)}$ to charge energy $e_{\min}$, where $\Delta U(i,t)$ and $\Delta C(i,t)$ are the increase of energy utility and cost of charging sensor s_i to t respectively.

We first initialize $\mathbf{T}$ to a zero vector and E_s to a zero matrix, meaning no sensor is charged, and no energy is shared. A choice set $\mathbb{T}^{\text{choice}}$ is defined to store the charging choices of each sensor. The choice $\mathbf{t}_{i,t}^{\text{choice}}$ means give sensor s_i t units of energy $e_{\min}$, and i and t are the index of sensor and discretized energy charging level respectively. The choice set $\mathbb{T}^{\text{choice}}$ is initialized to $\mathbb{T}^{\text{choice}} = \left\{\mathbf{t}_{i,t}^{\text{choice}} \middle| i=1,2,\cdots,n; t=1,2,\cdots,n_T\right\}$. In every iteration, we choose the sensor s_i with the highest energy utility ratio $\frac{\Delta U(i,t)}{\Delta C(i,t)}$ to charge energy $te_{\min}$, where $\Delta U(i,t)$ and $\Delta C(i,t)$ are the increase of energy utility and cost of update choice $\mathbf{t}_{i,t}^{\text{choice}}$ respectively, and we use a learning-based way to solve the TSP of WCV.

Notice that to update $\Delta C(i,t)$, a TSP must be solved. Here, we adopted a deep learning-based method proposed by W. Kool *et al.* [19] that combines attention mechanism and reinforcement learning. This algorithm uses a graph neural network (GNN) to encode the information of nodes and edges, and a pointer network to output a sequence of nodes representing the solution of TSP. The model is trained by maximizing the expected reward, which is the negative value of the TSP path length. This method performs comparable to heuristic algorithms and can provide solutions quickly.

During the iterations, the charging choice is removed from $\mathbb{T}^{\text{choice}}$ for two conditions:

- **Rule 1:** The cost of WCV exceeds $C_{\max}$ for update choice vector T' or the marginal energy utility of WCV is zero for update energy sharing matrix E'_s when using the charging choice $\mathbf{t}_{i,t}^{\text{choice}}$ to update T and E_s. (Line 9)
- **Rule 2:** If charging choice $\mathbf{t}_{i,t}^{\text{choice}}$ is chosen in the previous iteration, then the charging choice $\mathbf{t}_{i,j}^{\text{choice}}(j = 1 \cdots t)$ is removed from $\mathbb{T}^{\text{choice}}$. In other words, if sensor s_i is charged t units of energy $e_{\min}$, then we do not consider charging less energy to sensor s_i in further iterations. (Line 13)

While all the charging choices are removed from $\mathbb{T}^{\text{choice}}$, we get the optimal charging subset X and charging energy E_c.

The complexity of this algorithm is $O(n^2 n_T^2)$. Notice there is a TSP problem (Line 8), which is the basic calculation unit of the algorithm. Considering Line 13, rule 2 guarantees that at least one choice will be removed after one loop. Since we make a $\mathbb{T}^{\text{choice}}$ with nn_T choices (Line 1), The time complexity of algorithm is $O(n^2 n_T^2)$.

IV. Theoretical Analysis

In this section, we give the theoretical analysis of the performance of RMES. We will present a series of lemmas to conclude the $\frac{1}{8}(1 - 1/e)$ approximation ratio of RMES.

We first introduce how to express and view T in a set way. As the definition of T shows: a choice T means the charging choice $E_c = \{T_1 e_{\min}, T_2 e_{\min}, \cdots, T_n e_{\min}\}$. From another point of view, if we divide a sensor s_i to n_t pieces and perform a full-or-none charging with $e_{\min}$, then T_i means to charge T_i pieces of s_i. So we can introduce some set-like definitions of T for easier analysis:

1) $\mathbf{T}_1 \subseteq \mathbf{T}_2$ means for $\mathbf{T}_1 = \left\{T_1^1, T_2^1, \cdots, T_n^1\right\}$ and $\mathbf{T}_2 = \left\{T_1^2, T_2^2, \cdots, T_n^2\right\}$, $T_i^1 \leq T_i^2$ for $\forall i$, which means that T_i^1 pieces of s_i covers T_i^2 pieces of s_i.
2) $\mathbf{T} \cup e_{\min}^i$ means to update $\mathbf{T} = \{T_1, \cdots, T_i, \cdots, T_n\}$ to $\mathbf{T} \cup e_{\min}^i = \{T_1, \cdots, T_i + 1, \cdots, T_n\}$, which means to select one more piece of s_i to charge.

A. Analysis for Energy Sharing Strategy Property

Definition 3 (Nonnegativity, monotonicity, and submodularity)**.** *Let $\mathbf{T}_C$ be the full charge strategy, i.e., $\mathbf{T}_C = \{n_T, \cdots, n_T\}$. The $\mathcal{U}(\mathbf{T})$ is nonnegative, monotone, and submodular for T if and only if:*

- ***Nonnegativity***:$\mathcal{U}(\mathbf{T}) \geq 0$ *for* $\forall \mathbf{T} \subseteq \mathbf{T}_C$.
- ***Monotonicity***: $\mathcal{U}(\mathbf{T} \cup e_{\min}^i) \geq \mathcal{U}(\mathbf{T})$ *for* $\forall \mathbf{T} \subset \mathbf{T}_C$ *and* $\forall i$.
- ***Submodularity***: $\mathcal{U}(\mathbf{T}_1 + e_{\min}^i) - \mathcal{U}(\mathbf{T}_1) \geq \mathcal{U}(\mathbf{T}_2 + e_{\min}^i) - \mathcal{U}(\mathbf{T}_2)$ *for* $\forall \mathbf{T}_1 \subseteq \mathbf{T}_2 \subset \mathbf{T}_C$ *and* $\forall i$.

Lemma 1. *Energy utility $\mathcal{U}(\mathbf{T})$ is nonnegative, monotone, and submodular for T.*

The nonnegativity is proved from the definition of the energy utility function. To prove monotonicity, we decompose the energy utility into two parts, sharing and base, and show that adding energy to any charging choice will increase both of these parts, leading to an overall increase in the energy utility. Finally, to prove submodularity, we use a greedy sharing strategy and show that adding energy to a smaller charging

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choice will always have a greater marginal effect than adding energy to a larger charging choice.

B. Analysis for Charging Discretization

To deal with continuous charging, we adopt a charging energy discretization mechanism. In this subsection, we give the gap between our solution to the optimal continuous one.

Lemma 2. *When the energy capacity of the mobile charger satisfies that $C_{\max} > \alpha n \times 2\sqrt{2}d$, the optimal energy utility after charging energy discretization achieves at least $1/2$ of the optimal energy utility in the continuous case.*

The proof can be obtained by rounding down the energy charged to each sensor in the continuous optimal solution to a multiple of $e_{\min}$, and adding a feasible discrete solution where each sensor is charged with $e_{\min}$ amount of energy.

C. Analysis for Energy Sharing Strategy Gap

In this subsection, we analyze the gap between our greedy energy sharing strategy and the optimal sharing strategy. According to the previous analysis, we have $U(X) = U(T, E_s)$. Let P' be our greedy sharing strategy and P^* be the optimal, and energy utility under P' be $U(T, E'_s)$ and energy utility under P^* be $U(T, E^*_s)$.

Lemma 3. *For any selected sensor set X with their charging energy, the overall utility achieved by our greedy sharing strategy P' in $U(T, E'_s)$ can achieve at least $1/2$ of the optimal energy utility in $U(T, E^*_s)$ under optimal sharing strategy P^*.*

The proof is done by introducing an auxiliary formula and comparing the marginal energy utilities for different policies, including the proposed strategy, the optimal strategy, and some special policies. Based on these comparisons, it is concluded that the achieved energy utility with the proposed strategy is at least as good as that in the optimal policy, leading to the conclusion that the proposed strategy can achieve at least half of optimal energy utility.

D. Approximation Ratio Analysis

Theorem 2. *If energy capacity of mobile charger satisfies that $B > \alpha n \times 2\sqrt{2}d$, the proposed algorithm achieves $\frac{1}{8}(1-1/e)$ approximation ratio.*

Proof: In Lemma 1, we prove that utility sharing function is nonnegative, monotone, and submodular. Hence, we transform the joint optimization problem into a submodular maximization problem with a general routing constraint. Thus, referring to [18], our Cost-Efficient algorithm, where the nearest-neighbor rule is applied to calculate the traveling cost, would achieve $\frac{1}{2}(1 - 1/e)$ bi-criterion approximation ratio with a slightly relaxed budget constraint. The quality of the approximated cost function $C(\mathbf{T})$ in Equation (1) has a significant impact on the relaxed degree.

Furthermore, we prove the $1/2$ gap between discrete and continuous solutions in Lemma 2, and the $1/2$ approximation ratio of our greedy energy sharing strategy compared with the optimal in Lemma 3. Hence, combine all bounds above and we can obtain $\frac{1}{8}(1-1/e)$ approximation ratio of our solution. Besides, proof of the time complexity of our algorithm is omitted here due to the space limit. ■

V. EXPERIMENTS AND RESULTS

In order to verify the performance of our algorithm, we conducted experiments and changed the energy limit of the WCV and the number of sensors. We also selected a series of classic algorithms, such as the greedy algorithm, clustering algorithm, and TSP only, to compare with our algorithm. Each instance uses four cores of Intel Xeon ICX Platinum 8358.

A. Evaluation Setup

TABLE I
DEFAULT SETTING OF PARAMETERS

Parameter	Default Value
sensing region area $d \times d$	200×200
WCV maximum energy cost $C_{\max}$	27000
number of sensors n	150
unit traveling cost α	15
energy demand e_i range	$[20, 100]$
sharing energy loss coefficient p	0.001
sharing energy distance limit	15
WCV charge efficiency range	$[0.5, 0.7]$
discretization factor n_T	25
WCV charge limit	500

The default setting of parameters is shown in Table I.

B. Baseline Methods

Although no algorithm is completely compatible with the JOIN-ME problem, some works are similar to ours. We select the following algorithms for comparison:

a) **Utility Greedy:** This algorithm is similar to our algorithm, but it only considers the energy utility of the sensors. It is a greedy algorithm that iteratively selects the sensor with the highest energy utility $\Delta U(i, t)$ and assigns it to the WCV.

b) **Clustering Method:** This algorithm is a clustering algorithm that divides the sensors into several clusters. Many studies have shown that the clustering method can achieve a good result [4], [20]–[22]. We use the DBSCAN algorithm to cluster the sensors [23]. The sensor with the highest energy need is selected to charge to match our capability-sharing strategy. To calculate how much energy the WCV should charge, we give every cluster a weight w_i according to the energy need of the cluster:

$$w_{\text{cluster i}} = \frac{\sum_{j\in\text{cluster}} e_j/\eta_{\text{charged sensor},j}}{\eta_{\text{charged sensor}}}. \tag{8}$$

Then we calculate the energy for the WCV to visit all the sensors need to chargeC_{travel}, and allocate the rest of the energy to the sensors according to the weight w_i:

$$x_{\text{charged sensor}} = (C_{\max} - C_{\text{travel}})\frac{w_{\text{cluster}}\eta_{\text{charged sensor}}}{\sum_{i\in\text{cluster}} w_i}. \tag{9}$$

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*c) **TSP Only:*** In some studies, the energy sharing is not considered [24], [25]. The TSP only means the sharing efficiency is 0 and all other parameters are the same as our RMES algorithm.

*d) **One Hop:*** In this algorithm, we set the maximum energy sharing efficiency $\eta_{\max}$ as:

$$\eta_{i,j}^{\max} = \eta_{i,j} \quad \forall i, j, \tag{10}$$

and all other parameters are the same as RMES. This algorithm is intermediate between TSP Only and RMES.

C. RMES performance analysis

In this section, we will analyze the performance of the RMES algorithm. We will compare the RMES algorithm with other algorithms in Section V-B.

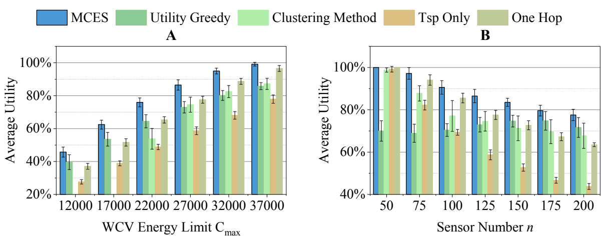

Fig. 2. The average energy utility under different map settings and algorithms. In subfigure (A), we change the WCV energy limit $C_{\max}$ from 12000 to 37000; In subfigure (B), we change the sensor number n from 50 to 200. Each situation is repeated 10 times with different random seeds.

As shown in Figure 2, we changed the sensor numbers n and the WCV energy limit $C_{\max}$, and compared the average energy utility. In Figure 2(A), we find the RMES algorithm outperforms other baselines, and the results are very stable for different random seeds. Notice that the clustering method can only be used when there is enough energy for touring all the clusters, so when the WCV Energy Limit $C_{\max}$ is below 22000, the clustering method cannot be used. In Figure 2(B), we still find that the RMES algorithm outperforms other baselines. The utility greedy method does not perform well when the sensor is sparse because the WCV does not consider the energy cost of the WCV. When the sensor number grows, the TSP Only method is badly degraded because it takes a long time to visit all the sensors.

The time for a solution is also recorded for analyzing the algorithm performance, shown in Figure 3. Since there are caching mechanisms for TSP-solving and energy sharing solving, and the $\mathbb{T}^{\text{choice}}$ can be removed for different reasons, the actual solving time is less than we estimate. Notice that the clustering method can be done at a very fast speed because is a practical solution, which is different from the searching method. In Figure 2(A), we find the solving time has a linear relationship with the WCV energy limit $C_{\max}$ because the WCV energy limit $C_{\max}$ is a bound for the search space. In Figure 2(B), we find that as the sensor number n grows, the solution time grows up in a linear way, which illustrates that our algorithm is available in large-scale solution.

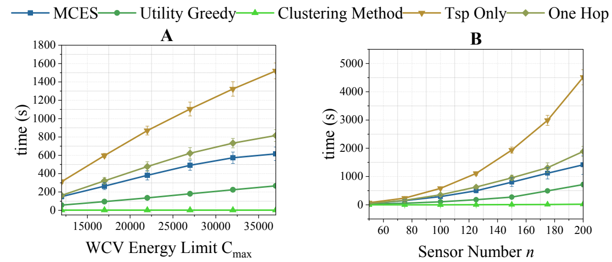

Fig. 3. The time for a single solution. In subfigure (A), we change the WCV energy limit $C_{\max}$ from 12000 to 37000; In subfigure (B), we change the sensor number n from 50 to 200. Each situation is repeated 10 times with different random seeds.

VI. Related Work

The optimization goal of the WSRNs can be generally divided into three categories, including improving the energy efficiency of mobile chargers [1], [12], [17], [26], minimizing charging delay [27]–[29] and optimizing network performance [30]–[32]. Our work aims to improve energy efficiency of WCV, a type of system in the first category. Goal is to maximize energy ratio for stable WSRN operation.

There are many scenarios of the WSRNs. Many works [12], [13] analyzed the WSRNs without sharing, and some works use multi WCVs to charge the sensors [4], [33]. Scenarios based on wireless bundle charging have been extensively studied over the years [2], [20], [32], [34]–[36], where the WCV can charge multiple sensors at the same time. Our work solves the scenario that the sensors can share energy, which is more complex because the geometry properties can not be used to solve the problem.

Convex-hull optimization, heuristic, geometry-based, and clustering-based methods are common methods for solving WSRN problems. Due to the relatively good geometric properties of wireless charging, many works [2], [35], [36] focus on geometric coverage optimization as the main solution strategy. Based on these studies, how to solve the problem that geometric properties are not highly correlated and the learning-based method is ineffective is a problem that needs to be solved in our work.

VII. Conclusion

RMES is a novel algorithm proposed for jointly optimizing mobile charging and energy sharing for rechargeable sensors in wireless networks. A greedy energy sharing strategy is constructed for multi-hop communication, and a charging energy discretization method is proposed for continuous charging, providing sensors with multiple charging choices. The RMES algorithm achieves an approximation ratio of $\frac{1}{8}(1 - 1/e)$ by formulating the problem into a submodular optimization problem with a general routing constraint. Theoretical analysis and simulation experiments demonstrate the effectiveness of the proposed algorithm in improving reliability and longevity of WSN, paving the way for advancements in areas such as environmental monitoring and crowd sensing.

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